Mode Propagation Through a Step Discontinuity in Dielectric Planar Waveguide

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Abstract—This paper presents two methods for dealing with wave propagation through a dielectric step discontinuity at normal incidence. One method helps to accelerate the convergence of solutions, especially for the TM-mode problem, and the other treats efficiently the continuous mode spectrum by introducing the Legendre transform in the case of open waveguides. As for the former, the singular fields around the dielectric edges are introduced in terms of direct use of their functional forms to the boundary condition, which is fulfilled in the sense of least squares. As for the latter, the expansion in terms of the Legendre functions is performed for optimally divided ranges of a continuous spectrum. A number of numerical examples prove that the methods presented herein are quite powerful for solving the TM-mode discontinuity problems in dielectric waveguides of both closed and open types.

I. Introduction

DISCONTINUITY problems in dielectric waveguides of both closed and open types play an important role in practical applications such as finite cascades of interacting step discontinuities, isolated discontinuities, etc., in integrated circuits ranging from microwave to optical frequencies. As for the finite cascades, they often appear as partial corrugations or gratings on dielectric waveguides for use in certain sophisticated components for integrated optics (e.g., the Bragg deflector [1] and the grating demultiplexer [2]) and also for similar components in the microwave to short millimeter-wave ranges (e.g., the grating filter and the leaky-wave antenna [3]).

In a previous paper [4], we described a microwave network approach to analyze such finite cascades, where the essential problems are that of the step discontinuity, upon which a surface wave impinges not normally but obliquely; in addition, the continuous mode spectrum must be taken into account in the case of open waveguide. As is well known [5], a TE or TM mode incident obliquely on a step discontinuity produces not only a reflected and a transmitted mode of its own type, but also excites a reflected and a transmitted mode of the other type in polarization.

For solving such a problem, one can immediately think of applying the mode-matching method in which two sets of normal modes in different kinds of waveguides are matched at the discontinuity plane. Certainly, the mode-matching method has been used straightforwardly in a great number of published papers [6]–[22]. However, direct application of this method for TM-mode discontinuity problems often suffers from a slow rate of convergence which may lead to inaccurate results. Therefore, we have to develop an effective method to find the solutions for the TM problem as well as for the TE problem, with rapid convergence, which leads to the identical degree in the accuracy of solutions for both types of incident mode.

As an intermediate step toward the oblique incident problem, this paper deals with the step discontinuity in two-dimensional planar dielectric waveguides of both closed and open types (as shown in Fig. 1) and considers the normal incidence of both mode types separately. For the TE-mode incidence, the convergence is usually very fast and it is easy to attain an accuracy of order 10^{-5} in the power conservation [19]–[22]. On the other hand, a slow rate of convergence in the case of TM-mode incidence often results from the singular behavior of the field at dielectric edges. Many papers (for example, [6], [11], [13], [17]–[22]) have indeed discussed the TM-mode problem, but a few [13], [19]–[22] have discussed this problem by taking account of the edge effect. Our present interest appropriately has close relation to Vassallo's paper [13] in which he presented a method based on the direct application of the Meixner's edge condition [23], which might improve the convergence of solutions for a planar dielectric waveguide of closed type. Since his approach expands the presupposed function of singular fields into the normal modes of the waveguide under consideration, it is necessary to take a large number of expansion terms even in case of a weak singularity. As for this problem, our paper can be considered an extension of Vassallo's work, but there is a distinct difference, which will be pointed out in Section II. A similar discussion on the edge condition, but in a different problem, can be found in [24].

On the other hand, for the discontinuity problem in open dielectric waveguides, one must always consider the appreciable coupling between the discrete surface-wave modes and waves with the continuous spectrum besides the edge singularity. Owing to the presence of this continuous spectrum, the discrete mode matching is intrinsically ill-suited for dealing with discontinuity problems in open waveguide. It is customary, however, in this class of problems to discretize the continuous spectrum by introducing

Manuscript received July 23, 1985; revised September 5, 1985.
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IEEE Log Number 8406474.

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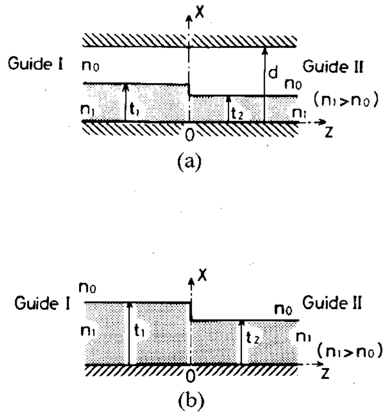

Fig. 1. Step-discontinuity configurations in planar dielectric waveguides of (a) closed type and (b) open type.

the Laguerre transform for the wavenumber [11]. This transform is indeed powerful enough to get a series with good convergence from so-called good functions, behaving well over the entire range of the continuous spectrum; in usual step discontinuities, however, most of the energy carried by an incident surface wave will couple strongly to a part of the continuous spectrum in a limited narrow range. In such a case, the Laguerre transform often causes the convergence difficulty. Employment of the Laguerre polynomials in a Ritz–Galerkin variational solution is another approach proposed by Rozzi [16]. Although this approach is successfully applied to a TE-wave discontinuity problem, it does not appear to be as readily applied to the TM-wave case.

To circumvent such inherent difficulties associated with the previous approaches, we propose an alternative method [19]–[22] to discretize efficiently the continuous spectrum in Section III.

II. Discontinuity in a Dielectric Waveguide of Closed Type

A. Analysis

Let us first consider a step discontinuity in a parallel-plate waveguide that is partially dielectric filled as seen in Fig. 1(a). The dielectric planar waveguide I, on the left-hand side has the thickness t_1, and the planar guide II, on the right-hand side, has the thickness t_2 ($<t_1$). We have here two dielectric edges along the y-axis at $x=t_1$ and t_2 at $z=0$, the effects of which should be taken into account carefully in case of TM-mode incidence. Since there is no longer any edge effect in the case of TE-mode incidence, the rapid convergence of solutions can be easily achieved even by means of the usual mode-matching method. Hereafter, our interest is concentrated on the fact that the qth TM-mode is incident normally onto the step discontinuity from the left-hand side of the structure.

The usual approach, which approximates the field in each guide by the mere truncation of an infinite series of orthonormal modal functions, solves the boundary problem by means of mode matching. One thereby misses some important information on the edge effect that is connected with neglected higher order modal functions, and suffers from a slow rate of convergence.

To recover such information, Vassallo [13] has described a method based on the direct application of the Meixner's edge condition [23], which considers the singular electric field of the order r^{γ} near edges, where r is the distance from an edge and $-1/2<\gamma<0$. Following his process, such singular fields around the edges are also expanded into a series of modal functions in the waveguide under consideration. It will be desirable to quote here [13, eq. (5)] in terms of our notations to show distinctly the difference of our approach from his:

$$T_m=\sum_{n=1}^{N} V_{mn}(\delta_{nq}+R_n)+\bar{R}\sum_{n>N} V_{mn}f(n) \tag{1}$$

where T_m means the amplitude of the mth mode excited in one guide when the qth mode, expressed by the Kronecher delta function, is incident to a discontinuity plane from another guide. V_{mn} are defined by integrals on the modal functions, while R_n and $\bar{R}$ are the unknown coefficients to be solved. The function $f(n)$ is written in terms of both the nth modal function and the field singularity term, as discussed in [13, Appendix A].

Equation (1) is indeed a better approximation to the true T_m than the mere truncation expressed simply by the first term in the right-hand side of (1), but it is clear from the last term in the right-hand side of (1) that this approximation still depends on the complete modal expansion of the singular field. Therefore, a good convergence and a satisfactory accuracy in solutions may be obtained only when summing up the series including the term $f(n)$ in (1) is possible for a remarkably large number of terms. However, it is reported [13] that there is difficulty in the summation of such a series especially in the case of weak singularity for a dielectric step, and little improvement is thereby achieved in convergence.

To overcome this difficulty, our approach presented here recovers the information of the edge effect not in terms of a modal expansion of the singular fields, but in terms of a direct use of its functional form. Returning now to the problem of Fig. 1(a), let us assume the x component of the singular electric fields locally bounded around $x=t_p$ by $|x-t_p|^{\gamma_p}$, ($p=1,2$ and $-1/2<\gamma_p<0$), and approximate the E_x component by[1]

$$\begin{aligned}E_x^{\mathrm{I}}(N)=&\sum_{n=0}^{N}(\delta_{nq}+R_n)e_{xn}^{\mathrm{I}}(x)\\&+R'\left\{|x-t_1|^{\gamma_1}-\sum_{n=0}^{N}A'_n e_{xn}^{\mathrm{I}}(x)\right\}\\&+R''\left\{|x-t_2|^{\gamma_2}-\sum_{n=0}^{N}A''_n e_{xn}^{\mathrm{I}}(x)\right\}\end{aligned} \tag{2}$$

[1]Expressions (2) and (3) are indeed inaccurate for the waveguide fields away from the dielectric edges, but the r^{γ} term has significant behavior just around the edges and rapidly goes to zero away from them. Therefore, the magnitude of this term distant from them contributes to the total fields with negligibly small amounts, and these expressions on the discontinuity plane will be acceptable as the first trial fields.

$$E_x^{\mathrm{II}}(M) = \sum_{m=0}^{M} T_m e_{xm}^{\mathrm{II}}(x) + T'\left\{ |x-t_1|^{\gamma_1} - \sum_{m=0}^{M} B'_m e_{xm}^{\mathrm{II}}(x) \right\} + T''\left\{ |x-t_2|^{\gamma_2} - \sum_{m=0}^{M} B''_m e_{xm}^{\mathrm{II}}(x) \right\} \quad (3)$$

where $e_{xj}^{i}(x)$ is the x component of the orthonormal modal functions of the jth mode in the guide i, and $R_n, R', R'', T_m, T', T''$ are unknown coefficients to be determined through the boundary condition.

The known amplitudes A'_n, A''_n, B'_m, and B''_m indicate the modal expansion coefficients of the singular fields for the orders $n \leqslant N$ and $m \leqslant M$, and are given by

$$A'_n = \int_0^d |x-t_1|^{\gamma_1} h_{yn}^{\mathrm{I}*}(x)\,dx$$

$$A''_n = \int_0^d |x-t_2|^{\gamma_2} h_{yn}^{\mathrm{I}*}(x)\,dx \quad (4)$$

$$B'_m = \int_0^d |x-t_1|^{\gamma_1} h_{ym}^{\mathrm{II}*}(x)\,dx$$

$$B''_m = \int_0^d |x-t_2|^{\gamma_2} h_{ym}^{\mathrm{II}*}(x)\,dx \quad (5)$$

where $h_{yj}^{i}(x)$ is the y component of the magnetic field corresponding to $e_{xj}^{i}(x)$ and the symbol * indicates the complex conjugate.

It should be appreciated here that, after decomposing the singular field into the contributions of the normal modes below the order N or M and the rest, Vassallo's approach still employs the latter contribution in terms of modal series, while ours introduces a contribution by subtracting the former contribution from the functional form itself of the singular field as seen in (2) and (3). Therefore, our approach has only to calculate a small number of amplitudes of (4) and (5) for $n \leqslant N$ and $m \leqslant M$, respectively, and does not encounter the difficulty that Vassallo's does.

So far, emphasis has been placed on the electric field. Next, let us mention briefly the way of treating the magnetic fields H_y^i, $(i = \mathrm{I}, \mathrm{II})$ around the dielectric edges. The singular electric field of order r^{γ}, if inserted in Maxwell's equation, yields a constituent of higher order $r^{\gamma+1}$ in the magnetic field, the amplitude of which is finite everywhere. Therefore, such a constituent has little influence on the rapid convergence, and is neglected for the approximated magnetic fields $H_y^{\mathrm{I}}(N)$ and $H_y^{\mathrm{II}}(M)$ in the present approach.

Finally, let us consider the boundary condition on the discontinuity plane at $z = 0$. Although the rigorous conditions are $E_x^{\mathrm{I}} = E_x^{\mathrm{II}}$ and $H_y^{\mathrm{I}} = H_y^{\mathrm{II}}$, the approximated fields $E_x^{\mathrm{I}}(N)$, $E_x^{\mathrm{II}}(M)$, $H_y^{\mathrm{I}}(N)$, and $H_y^{\mathrm{II}}(M)$ of (2) and (3) never satisfy the above type of boundary conditions. We therefore fit the approximated fields to the boundary conditions in the sense of least-squares [25]. For this purpose, we define the mean-square error ϵ for the boundary conditions, defined by the following equation:

$$\epsilon = \frac{1}{2}\left(\frac{\int_0^d |E_{\mathrm{tan}}^{\mathrm{I}} - E_{\mathrm{tan}}^{\mathrm{II}}|^2\,dx}{\int_0^d |e_{\mathrm{in}}|^2\,dx} + \frac{\int_0^d |H_{\mathrm{tan}}^{\mathrm{I}} - H_{\mathrm{tan}}^{\mathrm{II}}|^2\,dx}{\int_0^d |h_{\mathrm{in}}|^2\,dx} \right). \quad (6)$$

In the present problem, we ought to regard $E_x^{\mathrm{I}}(N)$, $E_x^{\mathrm{II}}(M)$, $H_y^{\mathrm{I}}(N)$, $H_y^{\mathrm{II}}(M)$, $e_{xq}^{\mathrm{I}}(x)$, and $h_{yq}^{\mathrm{I}}(x)$ as $E_{\mathrm{tan}}^{\mathrm{I}}$, $E_{\mathrm{tan}}^{\mathrm{II}}$, $H_{\mathrm{tan}}^{\mathrm{I}}$, $H_{\mathrm{tan}}^{\mathrm{II}}$, e_{in}, and h_{in}, respectively. This error is a function of R_n, T_m, R', R'', T', and T''. We then minimize ϵ with respect to these unknown coefficients, and solve for them by the same procedures as described in [26]. All of the numerical results which will appear in this paper are obtained considering this type of method of field matching.

It is interesting to note that Andersen *et al.* [27] have pointed out that the form of Meixner's solution in the time-varying case is not always correct for any configuration of dielectric edges and the relevant results may be obtained from the static case. Our cases treat the two dielectric edges which exist close to each other in a closed waveguide. Therefore, we think that it is difficult to find out the correct γ_p values for our case from the Meixner's approach, and our method regards the power indices γ_p $(p = 1, 2)$ of (2) and (3) still as two more unknown variables when the error ϵ is minimized.

B. Numerical Results

In Fig. 1(a), we assume n_1 and n_0 to be 1.46 and 1.0, respectively. Let us consider a case for which the discontinuity is described by the parameters $t_1/t_2 = 1.2$, $d/t_1 = 2.0$, and $k_0 d = 5.0$. For this structure, the TM_0 and TM_1 modes are above cutoff in each guide; numerical discussions are performed for a typical case in which the fundamental TM_0 mode is incident normally to the step from the left-hand side of guide I. We therefore compute the reflected and transmitted powers of TM_0 and TM_1 modes, the degree of power conservation (total power), and the least mean-square error ϵ by considering a number of modes below cutoff.

Table I(a) indicates the results obtained when only the first terms of the right-hand side of (2) and (3) (i.e., the summation over the discrete normal modes N and M in regions I and II; the edge effect is neglected altogether) are considered in our procedures, whereas Table I(b) shows the results obtained by the same procedures as Table I(a), but considering all of the terms in (2) and (3). We can recognize clearly a remarkable difference in the approximations; the former barely ensures the power conservation of 100.000 percent at $N = 200$, while the latter easily attains the same degree of power conservation at just $N = 20$. Moreover, the mean-square error ϵ, less than 0.001 percent, is achieved with $N \geqslant 150$ for the former approximation and with $N \geqslant 15$ for the latter, respectively.

Such a dramatic decrease in the number N in the latter approximation, which considers the edge singularity, has a great value in simplifying the numerical calculations itself. We may thereby ensure that the method presented here is

TABLE I
REFLECTION, TRANSMISSION POWERS OF TM_0 AND TM_1 MODES, DEGREE OF POWER CONSERVATION (TOTAL POWER), AND LEAST MEAN-SQUARE ERROR CALCULATED FOR DIFFERENT NUMBER N OF THE EXPANSION TERMS (IN CASE OF FIG. 1(a))

N	Reflected Power [%]		Transmitted Power [%]		Total Power [%]	Error [%]
	TM_0 mode	TM_1 mode	TM_0 mode	TM_1 mode		
10	0.001	0.010	99.546	0.039	99.595	0 354
20	0.001	0.013	99.796	0.046	99.856	0.102
30	0.001	0.014	99.891	0.048	99.954	0.039
40	0.001	0.014	99.907	0.048	99.972	0.024
50	0.002	0.014	99.918	0.049	99.982	0.016
100	0.002	0.015	99.932	0.049	99.997	0.004
150	0.002	0.015	99.934	0.049	99.999	0.001
200	0.002	0.015	99.934	0.049	100.000	0.000
250	0.002	0.015	99.934	0.049	100.000	0.000

(a) Present approach considering no edge singularity.

N	Reflected Power [%]		Transmitted Power [%]		Total Power [%]	Error [%]
	TM_0 mode	TM_1 mode	TM_0 mode	TM_1 mode		
5	0.001	0.015	99.925	0.049	99.990	0.009
10	0.002	0.015	99.929	0.049	99.994	0.004
15	0.002	0.015	99.935	0.050	100.001	0.001
20	0.002	0.015	99.934	0.049	100.000	0.000
25	0.002	0.015	99.934	0.049	100.000	0.000
30	0.002	0.015	99.934	0.049	100.000	0.000

(b) Present approach considering edge singularity.

quite effective to attain a rapid convergence for the case of TM-mode incidence.

We conclude this section with a plot of the mean-square error ϵ as a function of N, the number of expansion terms. It is obvious in Fig. 2 that the improvement obtained by the method considering edge effects in terms of the functional forms is quite sufficient.

III. DISCONTINUITY IN A DIELECTRIC WAVEGUIDE OF OPEN TYPE

A. Analysis

Let us next consider a step discontinuity in a dielectric planar waveguide of open type as shown in Fig. 1(b). On an open dielectric waveguide, the non-surface-wave modes comprise a continuous spectrum, a part of which is radiative, while the rest is reactive. Therefore, one must always consider appreciable coupling between the discrete surface-wave modes and the waves with continuous spectrum besides the effect of edge singularity which has already been discussed in the previous section.

It is customary, however, in this class of problem to discretize the continuous spectrum by employing the Laguerre transform as mentioned before. This transform is useful for achieving the good convergence for so-called good spectral functions behaving well over the entire range of the continuous spectrum. In the usual step discontinuity, however, most of the energy of an incident surface-wave mode will couple strongly to the waves with the continuous spectrum in a limited narrow range of the radiation part. In such a case, it is quite difficult to get a rapid convergence of solutions by means of the Laguerre transform, even if a great number of the Gauss–Laguerre functions of higher order are taken into account.

We describe here a new way of overcoming this difficulty. Our motivation is try to introduce a more flexible transform for discretizing a continuous spectrum. The basic idea is to divide the continuous spectrum into three ranges: one corresponds to the radiation part, the second is an optimally-scaled extent of the reactive part, and the third, which will be disregarded here, is the rest of the reactive part. To follow this approach, we have only to discretize independently the spectrum in each range. To this end, one can employ the Legendre transform for which the normalized Legendre functions provide the complete set of basis functions in each bounded range.

Now, let us consider that the qth TM surface-wave mode is incident normally onto the step discontinuity from the left-hand side of Fig. 1(b). Let $e^i_{xj}(x)$, $h^i_{yj}(x)$, $e^i_x(x,\rho)$, and $h^i_y(x,\rho)$ be the orthonormal modal functions of the jth surface-wave mode and the continuous wave in the guide i, respectively [28]. ρ means the wavenumber of the continuous wave in the x direction outside the waveguide and covers all values from 0 to ∞. As ρ covers the range $0 \leqslant \rho \leqslant n_0 k_0$, wher $k_0 = 2\pi/\lambda_0$, the corresponding field becomes radiative, while ρ is also allowed to fall in the range $n_0 k_0 \leqslant \rho \leqslant \infty$, in which the field becomes evanescent along the z direction. Let us introduce a scale parameter α to divide the latter range of ρ between $n_0 k_0 \leqslant \rho \leqslant \alpha n_0 k_0$ and $\alpha n_0 k_0 \leqslant \rho \leqslant \infty$. If the parameter α is optimally defined, one may disregard the field in the latter range, which has no significant effect on the total field.

Assuming here that N and M surface-wave modes are supported as the discrete modes in guide I and II, respectively, the electric fields tangential to the discontinuity plane can be expressed as follows:

$$E^{\mathrm{I}}_x(N) = \sum_{n=0}^{N} (\delta_{nq} + R_n) e^{\mathrm{I}}_{xn}(x) + \int_0^{\alpha n_0 k_0} \phi^{\mathrm{I}}(\rho) e^{\mathrm{I}}_x(x,\rho)\, d\rho + R'\{e_{s_1}(x) - g^{\mathrm{I}}_1(x)\} + R''\{e_{s_2}(x) - g^{\mathrm{I}}_2(x)\} \tag{7}$$

$$E^{\mathrm{II}}_x(M) = \sum_{m=0}^{M} T_m e^{\mathrm{II}}_{xm}(x) + \int_0^{\alpha n_0 k_0} \phi^{\mathrm{II}}(\rho) e^{\mathrm{II}}_x(x,\rho)\, d\rho + T'\{e_{s_1}(x) - g^{\mathrm{II}}_1(x)\} + T''\{e_{s_2}(x) - g^{\mathrm{II}}_2(x)\} \tag{8}$$

where R_n, R', R'', T_m, T', and T'' are the unknown coefficients to be determined. $e_{sp}(x)$, $(p = 1,2)$ denote the x components of the singular fields around $x = t_p$; one type of trial functional forms for them is assumed as follows:

$$e_{sp}(x) = \begin{cases} |x - t_p|^{\gamma_p}, & x \leqslant 2t_p \\ t_p^{\gamma_p} \exp\{\gamma_p (x - 2t_p)/t_p\}, & x \geqslant 2t_p \end{cases} \tag{9}$$

where γ_p takes the values from $-1/2$ to 0 as mentioned before, and the decaying $e_{sp}(x)$ beyond $x = 2t_p$ is assumed so as to assure the convergence of integrations with respect to x. Since the singular field e_{sp} naturally includes the identical components with the first two terms of the right-

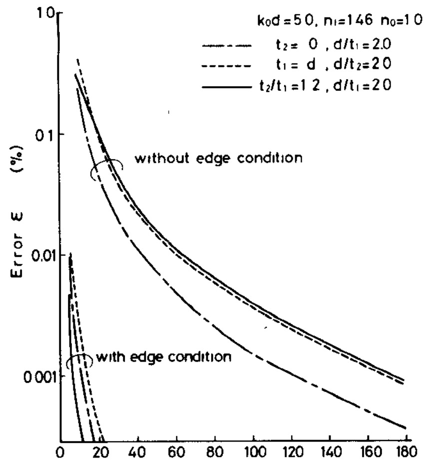

Fig. 2. Least mean-square error ϵ as a function of different number N of the expansion terms.

hand side of (7) or (8), it is needed to subtract these components g_p^i, $(i=\mathrm{I},\mathrm{II},\ p=1,2)$ from the singular fields. It is easily shown that g_p^{I} and g_p^{II} are calculated by

$$g_p^{\mathrm{I}}(x)=\sum_{n=0}^{N} e_{xn}^{\mathrm{I}}(x)\int_0^\infty e_{sp}(x)h_{yn}^{\mathrm{I}*}(x)\,dx + \int_0^{\alpha n_0k_0} e_x^{\mathrm{I}}(x,\rho)\,d\rho\int_0^\infty e_{sp}(x)h_y^{\mathrm{I}*}(x,\rho)\,dx \quad (10)$$

$$g_p^{\mathrm{II}}(x)=\sum_{m=0}^{M} e_{xm}^{\mathrm{II}}(x)\int_0^\infty e_{sp}(x)h_{ym}^{\mathrm{II}*}(x)\,dx + \int_0^{\alpha n_0k_0} e_x^{\mathrm{II}}(x,\rho)\,d\rho\int_0^\infty e_{sp}(x)h_y^{\mathrm{II}*}(x,\rho)\,dx. \quad (11)$$

Now, let us expand the spectral function $\phi^i(\rho)$ in (7) and (8) into the sum of proper functions defined in each range of ρ. An appropriate complete set of functions is provided by the normalized Legendre functions, and we denote the kth Legendre function by $P_k\{\xi(\rho)\}$ and $P_k\{\eta(\rho)\}$ in the bounded ranges $0\leqslant\rho\leqslant n_0k_0$ and $n_0k_0\leqslant\rho\leqslant\alpha n_0k_0$, respectively, where the functions $\xi(\rho)$ and $\eta(\rho)$ are given by

$$\xi(\rho)=\frac{2}{n_0k_0}\left(\rho-\frac{1}{2}n_0k_0\right)$$
$$\eta(\rho)=\frac{2}{(\alpha-1)n_0k_0}\left\{\rho-\frac{(\alpha+1)n_0k_0}{2}\right\} \quad (12)$$

because $P_k(x)$ is the orthonormal function defined in the range $|x|\leqslant 1$. The orthonormal nature of P_k leads to the following expansions holding for the continuous spectra (for example, $\phi^{\mathrm{I}}(\rho)$ and $\phi^{\mathrm{II}}(\rho)$ in the range $0\leqslant\rho\leqslant n_0k_0$):

$$\left.\begin{aligned}\phi^{\mathrm{I}}(\rho)&=\sum_{k=0}^{\infty}R_k'P_k\{\xi(\rho)\}\\ \Phi^{\mathrm{II}}(\rho)&=\sum_{l=0}^{\infty}T_l'P_l\{\xi(\rho)\}\end{aligned}\right\}\quad(0\le\rho\le n_0k_0)\quad(13)$$

where R_k' and T_l' are the additional unknown coefficients to be determined; these series will be truncated, in practice, by K_1 and L_1 terms, respectively. As a result, we can rewrite (7) and (8) as follows:

$$\begin{aligned}E_x^{\mathrm{I}}(N,K_1,K_2)=&\sum_{n=0}^{N}(\delta_{nq}+R_n)e_{xn}^{\mathrm{I}}(x)\\&+\sum_{k=0}^{K_1}R_k'\int_0^{n_0k_0}P_k\{\xi(\rho)\}e_x^{\mathrm{I}}(x,\rho)\,d\rho\\&+\sum_{k=0}^{K_2}R_k''\int_{n_0k_0}^{\alpha n_0k_0}\{\eta(\rho)\}e_x^{\mathrm{I}}(x,\rho)\,d\rho\\&+R'\{e_{s_1}(x)-g_1^{\mathrm{I}}(x)\}\\&+R''\{e_{s_2}(x)-g_2^{\mathrm{I}}(x)\}\end{aligned}\quad(14)$$

$$\begin{aligned}E_x^{\mathrm{II}}(M,L_1,L_2)=&\sum_{m=0}^{M}T_me_{xm}^{\mathrm{II}}(x)\\&+\sum_{l=0}^{L_1}T_l'\int_0^{n_0k_0}P_l\{\xi(\rho)\}e_x^{\mathrm{II}}(x,\rho)\,d\rho\\&+\sum_{l=0}^{L_2}T_l''\int_{n_0k_0}^{\alpha n_0k_0}P_l\{\eta(\rho)\}e_x^{\mathrm{II}}(x,\rho)\,d\rho\\&+T'\{e_{s_1}(x)-g_1^{\mathrm{II}}(x)\}\\&+T''\{e_{s_2}(x)-g_2^{\mathrm{II}}(x)\}.\end{aligned}\quad(15)$$

On the other hand, as mentioned in Section II-A, the singular electric field given by (9) yields a constituent in the magnetic field, the amplitude of which is finite everywhere. Assuming that such a constituent has little influence on the convergence, we approximate the magnetic fields $H_y^{\mathrm{I}}(N,K_1,K_2)$ and $H_y^{\mathrm{II}}(M,L_1,L_2)$ by those belonging to the first three terms of the right-hand side of (14) and (15). Consequently, in the present problem, we can solve the unknown coefficients by the same way as discussed in Section II-A, by regarding $E_x^{\mathrm{I}}(N,K_1,K_2)$, $E_x^{\mathrm{II}}(M,L_1,L_2)$, $H_y^{\mathrm{I}}(N,K_1,K_2)$, $H_y^{\mathrm{II}}(M,L_1,L_2)$, $e_{xq}^{\mathrm{I}}(x)$, and $h_{yq}^{\mathrm{I}}(x)$ as $E_{\tan}^{\mathrm{I}}$, $E_{\tan}^{\mathrm{II}}$, $H_{\tan}^{\mathrm{I}}$, $H_{\tan}^{\mathrm{II}}$, e_{in}, and h_{in} of (6), respectively.

B. Numerical Results

In Fig. 1(b), we assume $n_1=1.46$, $n_0=1.0$, $k_0t_1=2.5$, and $t_1/t_2=1.2$. For this structure, each guide supports TM_0 and TE_1 modes only as the discrete modes. But, we discuss here the case of each mode incidence normally to the step discontinuity, and the mode coupling between these modes does not occur [5]. First we discuss numerically the case of TM_0-mode incidence from the left-hand side of guide I. Therefore, we put $q=0$ and $N=M=0$ in (10), (11), (14), and (15). After assuming $K_1=K_2=L_1=L_2$ and the scale factor $\alpha=7$ in (14) and (15), we compute the reflection and transmission powers of the TM_0 surface-wave mode, the radiation power, the degree of power conservation (total power), and the least mean-square error ϵ. Table II(a) indicates the results obtained for an abbreviated case which employs only the first three terms of the right-hand side of (14) and (15) and disregards the edge effects expressed by the fourth and fifth terms. $K=K_1=$

TABLE II
REFLECTION, TRANSMISSION POWERS OF TM_0 SURFACE-WAVE MODE, RADIATION POWER, DEGREE OF POWER CONSERVATION (TOTAL POWER), AND LEAST MEAN-SQUARE ERROR FOR DIFFERENT NUMBER K OF THE EXPANSION TERMS OF THE LEGENDRE FUNCTIONS

K	Reflected Power(TM_0)	Transmitted Power(TM_0)	Radiation Power Reflected	Radiation Power Transmitted	Total Power [%]	Error [%]
1	0.000	98 384	0.027	0.069	98.481	1.107
2	0.000	98.483	0.024	0.072	98.579	1.019
3	0.000	98 474	0.019	0.062	98.555	0.976
4	0 000	98.807	0.006	0.080	98.889	0.753
5	0.000	99.053	0.010	0.070	99.134	0.659
6	0.001	99.569	0.015	0.070	99.655	0.435
7	0.001	99.531	0.018	0.068	99.617	0.356
8	0.001	99.530	0.019	0.068	99.618	0.329
9	0.001	99.530	0.019	0.068	99.618	0.326

(a) Present approach considering no edge singularity.

K	Reflected Power(TM_0)	Transmitted Power(TM_0)	Radiation Power Reflected	Radiation Power Transmitted	Total Power [%]	Error [%]
4	0.001	99.822	0.039	0.075	99.937	0.117
5	0.002	99 972	0.034	0.085	100.093	0.065
6	0.002	99.873	0.028	0.080	99.982	0.054
7	0.001	99.849	0.030	0.081	99.962	0.038
8	0.001	99.838	0.025	0.076	99.940	0.033
9	0.001	99.848	0.027	0.078	99.954	0.032

(b) Present approach considering edge singularity.

K_2 means the number of terms used in the Legendre expansion. For $K = 8$, the above quantities, which completely characterize the discontinuity, have reached their convergence values. However, the magnitudes obtained for the total power (99.6 percent) and the error ϵ (0.329 percent) are unsatisfactory for practical applications, especially in the cascade connection of such discontinuities, and also they deteriorate the confidence in the convergence values obtained.

The same quantities are now computed by the identical procedure, by taking account of the edge singularity. The results are shown in Table II(b). As expected, it is clear that this approximation improves the results shown in Table II(a) for the magnitudes of the total power and the error ϵ by one figure or more. Therefore, we may conclude that the results obtained by the last method will be more reliable than those of Table II(a), though imperfect convergence is seen in some of quantities of Table II(b) even at $K = 9$. However, the magnitude of fluctuations seems to be small enough so that the present approximation is justified in practice, and, on account of this, the following results are computed for $K = 9$.

Fig. 3 shows the reflection, transmission, and radiation powers as a function of t_2/t_1. The relative transmission power is 100 percent at $t_2/t_1 = 1.0$, as it should, since the discontinuity disappears. As t_2/t_1 decreases, the transmission power goes to zero, while the radiation power reaches almost 100 percent and the reflection power goes to its small limiting value, since the surface-wave mode is no longer guided in guide II for $t_2/t_1 = 0$. Fig. 4 shows the radiation patterns calculated by the steepest descent method [9], where the peak value is normalized to unity for each radiation pattern. The axes along $\theta = 0°$ and $90°$ coincide with the z and x directions, respectively. Since the TM_0 mode has the E_x component symmetric with respect to the $y-z$ plane at $x = 0$, the radiation occurs into the end-fire (z axis) direction for $t_2/t_1 = 0$. As t_2/t_1 increases, the angle θ_{max} of the radiation peak changes from zero to a limiting angle of elevation on account of the step discontinuity. Also, the complicated side lobes appear with increasing θ_{max}. This may be attributed to the edge effects.

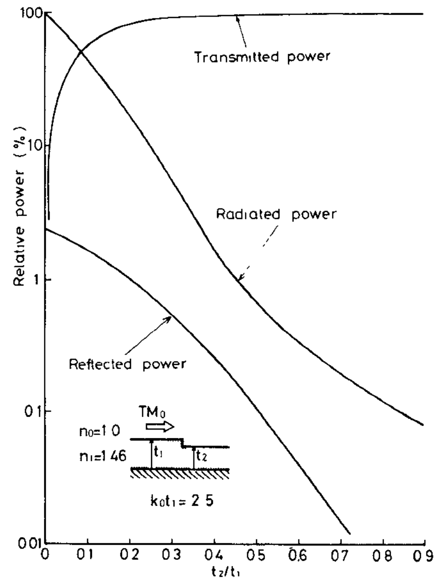

Fig. 3. Reflection, transmission, and radiation powers as a function of t_2/t_1, for the TM_0 mode incident from guide I.

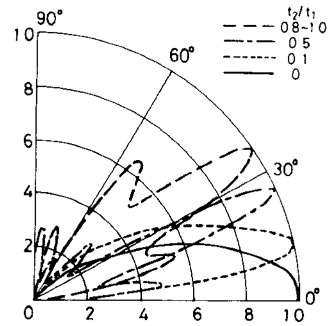

Fig. 4. Radiation patterns for different ratio t_2/t_1 as a parameter (in the case of Fig. 3).

Finally, for the sake of comparison, let us consider the case of a TE surface-wave mode. As shown in Fig. 1(b), the guide under consideration has a ground plane at the $y-z$ plane, so that the fundamental TE mode becomes a TE_1 mode. Therefore, in this case, q, n, and m should start from unity in (10), (11), (14), and (15), instead of from zero as seen in the TM incident case. As mentioned at the beginning of this section, the structure under consideration propagates only the TE_1 surface-wave mode, and we put $q = 1$ and $N = M = 1$. Table III indicates the results, which are calculated for the same structure as employed in Table II. This problem no longer poses any difficulty caused by the edge singularity. We thereby have only to follow the Legendre transform in the same fashion as mentioned in Section III-A. It is seen from Table III that the present approach easily ensures the power conservation of 99.999

TABLE III
REFLECTION, TRANSMISSION, POWERS OF TE_1 SURFACE-WAVE MODE, RADIATION POWER, DEGREE OF POWER CONSERVATION (TOTAL POWER), AND LEAST MEAN-SQUARE ERROR FOR DIFFERENT NUMBER K OF THE EXPANSION TERMS OF THE LEGENDRE FUNCTIONS

K	Reflected Power(TE_1)	Transmitted Power(TE_1)	Radiation Power		Total Power [%]	Error [%]
			Reflected	Transmitted		
1	0.108	98.925	0.040	0.220	99.293	0.740
2	0.133	99.012	0.018	0.478	99.621	0.382
3	0.113	99.122	0.005	0.534	99.774	0.231
4	0.111	99.207	0.008	0.550	99.876	0.128
5	0.109	99.269	0.006	0.552	99.937	0.067
6	0.110	99.307	0.007	0.554	99.979	0.023
7	0.111	99 326	0.007	0.555	99.999	0.004
8	0.111	99.326	0.007	0.555	99.999	0.002
9	0.111	99.326	0.007	0.555	99.999	0.001

This case does not encounter any edge effect.

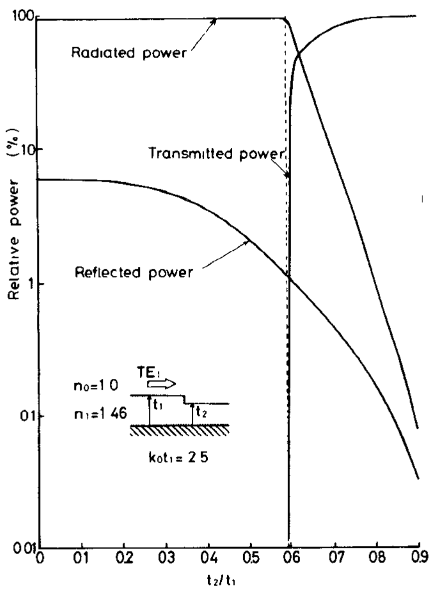

Fig. 5. Reflection, transmission, and radiation powers as a function of t_2/t_1, for the TE_1 mode incident from guide I.

percent and the least mean-square error ϵ less than 0.001 percent at $K=9$. Fig. 5 shows each power as a function of t_2/t_1 and the radiation patterns are shown in Fig. 6. Since the TE_1 surface-wave mode is a higher mode in the waveguide shown by the inset of Fig. 5, this mode in guide II becomes cutoff at $t_2/t_1 \cong 0.6$, which is shown by the dashed line in Fig. 5. Thus, in the cutoff region ($0 < t_2/t_1 \lesssim 0.6$), most of the incident power is radiated. Since the TE_1 mode has the E_y component antisymmetric with respect to the $y-z$ plane, radiation at $t_2/t_1 = 0$ occurs into an elevated angle ($\theta_{max} \cong 28°$), and as t_2/t_1 increases, θ_{max} decreases to its minimum value 10°, which occurs at the cutoff value $t_2/t_1 \cong 0.6$. Then, after going through a minimum, θ_{max} reaches a limiting angle $\theta_{max} \cong 18°$, as t_2/t_1 increases. As of now, the authors have no reasonable way to explain well these features physically.

IV. CONCLUSION

The step discontinuity in planar dielectric waveguides of both closed and open types has been treated for the TM-mode incidence as well as for the TE-mode incidence.

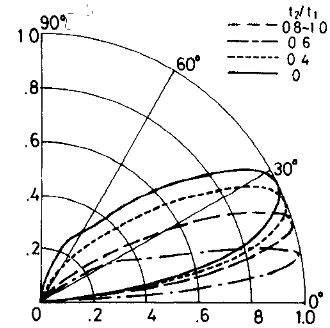

Fig 6. Radiation patterns for different ratio t_2/t_1 as a parameter (in the case of Fig. 5).

The authors have emphasized the discontinuity problem associated with the TM mode at normal incidence. As for the field singularity, the present approach first assumes the singular fields locally bounded around the dielectric edges and introduces such singular components into the field expression in terms of direct use of their functional forms, and finally fits the fields in the two guides at the discontinuity plane in the sense of least-squares. As for the continuous spectrum, we divide it into three ranges, one of which is disregarded here. Then the spectral function in each range is expanded in terms of the Legendre functions. A number of numerical results are presented for the TM-mode problem in comparison with the TE-mode problem. These results demonstrate that significant improvement in convergence and also in the accuracy of results are achieved even for the TM-mode case.

The same technique can also be applied to other important TM discontinuity problems in dielectric waveguides.

ACKNOWLEDGMENT

The authors are deeply indebted to Prof. K. Takiyama of Doshisha University for his useful advice and encouragement. The authors also wish to express their thanks to Prof. A. A. Oliner of the Polytechnic Institute of New York for his constant interest in the present work and his useful discussions. The authors thank the reviewers for their kind comments.

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